

Convex Hull Pricing for Demand Response in Electricity Markets

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Abstract—Dynamic pricing (a.k.a. real-time pricing) is a method of invoking a response in demand pricing electricity at hourly (or more often) intervals. Several studies have proposed dynamic pricing models that maximize the sum of the welfares of consumers and suppliers under the condition that the supply and demand are equal. They assume that the cost functions of suppliers are convex. In practice, however, they are not convex because of the startup costs of generators. On the other hand, many studies have taken startup costs into consideration for unit commitment problems (UCPs) with a fixed demand. The Lagrange multiplier of the UCP, called convex hull pricing (CHP), minimizes the uplift payment that is disadvantageous to suppliers. However, CHP has not been used in the context of demand response.

This paper presents a new dynamic pricing model based on CHP. We apply CHP approach invented for the UCP to a demand response market model, and theoretically show that the CHP is given by the Lagrange multiplier of a social welfare maximization problem whose objective function is represented as the sum of the customer's utility and supplier's profit. In addition, we solve the dual problem by using an iterative algorithm based on the subgradient method. Numerical simulations show that the prices determined by our algorithm give sufficiently small uplift payments in a realistic number of iterations.

I. INTRODUCTION

Dynamic pricing (a.k.a. real-time pricing) is a method of invoking demand response [1] by pricing at hourly (or more often) intervals. In contemporary U.S. structured wholesale markets and deregulated retail markets, several market operators (e.g. New York ISO [2]) have introduced dynamic pricing mechanisms based on demand response market models. A question arises in regard to such markets: what kind of pricing scheme should the market operator choose? Recently, several studies have proposed dynamic pricing models. In particular, Roozbehani et al. [3] proposed a nonlinear control model for changing electricity prices in a real-time market. They focused on the stability of the market and analysed their model using volatility measures. They showed that their model had stabilizing effects on the market. On the other hand, Miyano and Namerikawa [4] proposed a pricing model in a day-ahead market where the market operator makes the next day's hourly pricing decision. They studied ways of controlling the load level by using dynamic pricing. Their pricing algorithm is based on steepest descent and has a good controlling effect.

These papers proposed pricing models that maximize the sum of the welfares of producers and consumers under the

condition that supply and demand are equal. They assume that the cost functions of suppliers are convex. However, it is natural to assume a nonconvex and discontinuous cost function because of the startup costs of generators. Indeed, such a cost function is assumed within the setting of unit commitment problems (UCPs) [5], [6], [7], [8], [9]. A UCP is one to find the minimum cost of dispatching generators to meet a fixed electrical load. It does not consider dynamic demand; that is, it does not control the demand by changing the electricity price. Electricity pricing models that reflect startup costs have been studied in the context of UCP [5], [6], [7], [8], [9]. Because of startup costs, however, none of pricing models may be able to bridge the gap between the optimal profit and the supplier's actual profit. This gap is called the *uplift payment*, and it can be regarded as a measure of the supplier's disadvantage. Several pricing models have been studied in order to make the uplift payment small (See [5], [6], [7] for a comparison of these models). The most successful pricing model at reducing uplift payments is the convex hull pricing (CHP) (a.k.a. extended locational marginal pricing) proposed by Gribik et al. [6]. The authors theoretically showed that CHP minimizes the uplift payment. The uplift payments for owners of many generators tend to be relatively small and can often be ignored. However, this may not be true for small producers since startup costs occupy a large portion of the total cost of electricity generation.

This paper presents a new dynamic pricing model based on CHP. We applied a CHP approach invented for the UCP to a demand response market model; i.e., our market model considers *both* the demand response and the startup cost. First, we define a social welfare maximization problem which maximizes the sum of consumers' utility and suppliers' profit under the condition that supply and demand are equal, and then we theoretically show that the CHP is given by a solution of its dual problem, i.e., the Lagrange multiplier. This implies that our price minimizes the uplift payment for the equilibrium demand. In addition, we provide an iterative pricing algorithm based on a subgradient method to solve the dual problem. Numerical results show that our pricing schemes led to smaller uplift payments compared with standard pricing schemes. Since our pricing model has a nonsmooth objective function including 0-1 integer variables, it is difficult to be solved by exact optimization algorithms. However, in numerical experiments, the prices of our pricing algorithm lead to sufficiently small uplift payments in a realistic number of iterations.

The remainder of this paper is organized as follows.

Section II presents the setting of an electricity market. Section III defines the UCP and the convex hull price. Section IV introduces a new dynamic pricing model that includes a social-welfare maximization problem and UCP. We theoretically show that this pricing model leads to the convex hull price. A subgradient based method is applied to our model in Section V. Numerical simulation results are reported in Section VI. We give conclusions and list possible directions for research in Section VII.

In what follows, we denote column vectors in bold face, e.g., $\mathbf{x} \in \mathbb{R}^n$ whose i -th element is $x_i \in \mathbb{R}$ ($i = 1, 2, \dots, n$).

II. MARKET MODEL

We will begin by describing the electricity market model and an existing pricing model. An electricity market has three kinds of participants: consumers, suppliers, and an independent system operator (ISO). The suppliers (or consumers) decide their electric power production (or consumption) so as to maximize their profit (or utility, respectively) at a given price of electricity. The ISO is a non-profit institution that is independent of electric power companies. The ISO makes hourly (or more often) pricing decisions to balance supply and demand.

A. Supply and demand model

We assume a single representative supplier (or consumer) whose response represents the aggregate response of several suppliers (or consumers) and focus on a one-period model (i.e., a model with that does not take into account dynamical changes) in order to simplify our notation. Our model and algorithm can be extended to a multi-agent and multi-period model, as is done in [5]. Let $u : [0, \infty) \rightarrow [0, \infty)$ be the utility function of a representative consumer, which represents the dollar value of consuming electricity. Let $v : [0, \infty) \rightarrow [0, \infty)$ be the cost function of a representative supplier, which represents the dollar cost of producing electricity. We place some assumptions on these functions.

Assumption 1: The utility function u is class- \mathcal{C}^2 , monotonically increasing, and strictly concave on $[0, \infty)$. The cost function v is class- \mathcal{C}^2 , monotonically increasing, and strictly convex on $[0, \infty)$. (Examples of u and v are shown in [4].)

The consumer and supplier respectively maximize their utility and profit. Therefore, for a given electricity price $p > 0$, they determine their optimal electricity consumption $d(p) \geq 0$ and production $s(p) \geq 0$ by solving the following utility and profit maximization problems:

$$d(p) = \arg \max_{d \geq 0} u(d) - pd \quad (1)$$

$$= \max_{d \geq 0} \{0, \{d \mid \dot{u}(d) = p\}\}, \quad (2)$$

$$s(p) = \arg \max_{y \geq 0} py - v(y) \quad (3)$$

$$= \max_{y \geq 0} \{0, \{y \mid \dot{v}(y) = p\}\}.$$

(1) and (3) respectively corresponds to a consumer's and supplier's problem.

B. Pricing model

The ISO is an independent, nonprofit institution whose primary object is to match supply and demand. In the electricity

market, neither the consumer nor the supplier bid. Hence, the ISO matches the supply $s(p)$ and demand $d(p)$ by making an appropriate pricing decision p . Here, we make the following assumption.

Assumption 2: The utility function u of the consumer is unknown to the ISO. The cost function v of the supplier is not necessarily known to the ISO.

Accordingly, an electricity price p is determined through the following procedure:

- 1) The ISO sets a price p^k and sends it to the supplier and consumer.
- 2) The supplier and consumer make their operation plans, $s(p^k)$ and $d(p^k)$, and send them to the ISO.
- 3) If there is a gap between the levels of production $s(p^k)$ and consumption $d(p^k)$, the ISO assigns a new price p^{k+1} to manage the balance of supply and demand and sends p^{k+1} back to the supplier and consumer.
- 4) Steps 2 and 3 are repeated N times.

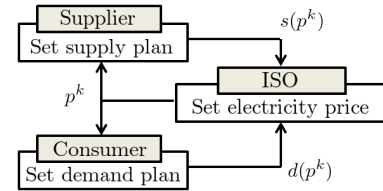


Fig. 1. Schematic views of the electricity market model.

Miyano and Namerikawa [4] proposed a pricing model based on a social-welfare maximization problem under the condition that the supply and demand are equal. Social welfare is defined as the sum of the consumer's profit (as in (1)) and the supplier's profit (as in (3)):

$$\{u(d) - pd\} + \{py - v(y)\}. \quad (4)$$

Here, we make the following assumptions in order to simplify the mathematical models.

Assumption 3:

- 1) Resistive losses in transmission and distribution lines can be ignored.
- 2) Line capacities are high enough so that congestion will not occur.
- 3) There are no reserve capacity constraints.

Under Assumption 3, the supply-demand balance is satisfied if and only if $d = y$. Thus, we obtain the following social-welfare maximization problem:

$$\max_{y \geq 0, d \geq 0} u(d) - v(y) \quad \text{s.t.} \quad d = y. \quad (5)$$

Or more simply,

$$\max_{d \geq 0} u(d) - v(d). \quad (6)$$

However, because of Assumption 2, the ISO cannot solve problem (5) (or (6)) directly. For this reason, the following partial Lagrangian dual problem of (5) is considered in [4]:

$$\min_{\lambda} \varphi(\lambda), \quad (7)$$

where

$$\begin{aligned}\varphi(\lambda) &= \max_{d \geq 0, y \geq 0} u(d) - v(y) + \lambda(y - d) \\ &= \max_{d \geq 0} \{u(d) - \lambda d\} + \max_{y \geq 0} \{\lambda y - v(y)\}.\end{aligned}\quad (8)$$

The optimal solution of (7) is called the Lagrange multiplier for (5). Since the maximization problems in (8) correspond to (1) and (3), the ISO can make the consumer and supplier solve (8) by sending λ to them. This mechanism allows the ISO to solve (7) by using the steepest descent method. (See [4] for details.)

Lemma 1 ([4]): Suppose that (d^*, y^*) is an optimal solution of (5), and λ^* is an optimal solution of (7). If Assumption 1–3 are satisfied, we obtain $d^* = d(\lambda^*)$ and $y^* = s(\lambda^*)$.

Lemma 1 states that if λ^* is chosen as a price, the demand $d(\lambda^*)$ and the supply $s(\lambda^*)$ maximize the social welfare (4). In other words, λ^* can be regarded as an optimal price.

These results assume that the cost function v is convex. Here, we consider an electricity market model with a non-convex cost function in the setting of the unit commitment problem (UCP) and provide an efficient pricing model for it.

III. UNIT COMMITMENT PROBLEM

We assume that the supplier has several different electricity generators, possibly with different generating costs and startup costs. The question for the supplier is how to operate electricity generators in order to supply a given scheduled demand y at the minimum cost. Here, we suppose that Assumption 3 is satisfied¹. Accordingly, the minimum cost function $v : [0, \infty) \rightarrow [0, \infty]$ can be described as follows:

$$v(y) := \begin{cases} \inf_{g, z} \sum_j C_j(g_j) + \sum_j S_j z_j \\ \text{s.t. } \sum_j g_j = y \\ m_j z_j \leq g_j \leq M_j z_j, \quad z_j \in \{0, 1\}, \forall j, \end{cases} \quad (9)$$

where C_j is a piece-wise linear generating cost function for an output g_j ; S_j denotes the startup cost; m_j and M_j denote the minimum and maximum outputs; and z_j represents a decision to commit generator j . Problem (9) is called a *unit commitment problem (UCP)*. The optimal value v of the UCP is characterized by lower semicontinuity, and it may be nonconvex. Several pricing models that reflect the startup costs have been developed in the setting of the UCPs, and some of them are compared in [5], [6], [7]. A specific example of a pricing model is described in the next section.

A. Convex hull price

A number of studies (e.g. [5], [6], [7], [8]) have tried to reduce the *uplift payment*, which is defined as follows.

Definition 1 (Uplift payment): Suppose that a demand y is given. The uplift payment $\Pi(p, y)$ for a price p is defined as follows:

$$\Pi(p, y) := \sup_x \{px - v(x)\} - \{py - v(y)\}.$$

¹We can apply the method proposed in this paper to a general model without Assumption 3 (for example, the model shown in [6]), though the resulting model is complicated.

The first term $\sup_x \{px - v(x)\}$ represents the maximum realizable profit with respect to the supply x under the price p , and the second term $\{py - v(y)\}$ represents the actual profit at p and given demand y . The uplift payment can be regarded as a cost incentive to make the supplier produce electricity to meet the scheduled demand y . Therefore, it is natural to find a price, p , that minimizes the uplift payment.

Such a price can be obtained by *convex hull pricing (CHP)* (a.k.a. the extended locational marginal pricing), as proposed in [6]. Before introducing CHP, we define the convex hull of the cost function $v(y)$:

Definition 2 (Convex hull of the cost function): The convex hull v^h of v is defined as

$$v^h(y) := \inf \{ \mu \mid (y, \mu) \in \text{conv}(\text{epi}(v)) \},$$

where $\text{conv}(A)$ is the convex hull of a set A , and $\text{epi}(v)$ is the epigraph of v .

The convex hull of v can be regarded as the largest convex function that is bounded above by v at any point in its domain. (A conceptual illustration is given in [10].) Now, we can define the CHP.

Definition 3 (Convex hull price): A convex hull price p^h is defined as the subgradient of the convex hull v^h of the cost function v at a given demand y , i.e.,

$$p^h \in \partial v^h(y).$$

The important property here is that the CHP minimizes the uplift payment. Gribik et al. theoretically proved this property in connection with duality theory. They also showed that the CHP can be obtained by solving a partial Lagrangian dual problem even though the explicit forms of v and its convex hull v^h are generally too complicated to compute. The next two propositions are from [6].

Proposition 1 ([6]): A convex hull price p^h minimizes the uplift payment $\Pi(\cdot, y)$ with the given demand y , i.e.,

$$p^h \in \arg \min_{p \geq 0} \Pi(p, y).$$

Proposition 2 ([6]): Suppose that λ^* is an optimal solution of the partial Lagrangian dual problem of (9), as defined below:

$$\max_{\lambda} \begin{cases} \min_{g, z} \sum_j C_j(g_j) + \sum_j S_j z_j + \lambda(y - \sum_j g_j) \\ \text{s.t. } m_j z_j \leq g_j \leq M_j z_j, \quad z_j \in \{0, 1\}, \forall j. \end{cases} \quad (10)$$

Then, λ^* is a convex hull price, i.e.,

$$\lambda^* \in \partial v^h(y).$$

Although Proposition 2 gives us a way to obtain the CHP, there still remains a difficulty in that (10) contains a mixed integer programming problem. Many researchers have studied algorithms for (10), e.g., [10], [11].

IV. PRICING MODEL

Here, we will add the minimum cost function v of (9) into the supply model (3) of the market model. We derive a

new ISO's social-welfare maximization problem from (5) as follows:

$$\begin{aligned} \max_{\mathbf{g}, \mathbf{z}, d} \quad & u(d) - \left\{ \sum_j C_j(g_j) + \sum_j S_j z_j \right\} \\ \text{s.t.} \quad & \sum_j g_j = d, \quad d \geq 0 \\ & z_j m_j \leq g_j \leq z_j M_j, \quad z_j \in \{0, 1\}, \quad \forall j, \end{aligned} \quad (11)$$

We will write this more simply, as (6). Since the ISO cannot solve (11) directly because of Assumption 2, we will instead consider a partial Lagrangian dual problem. For notational simplicity, let X denotes the feasible set for outputs \mathbf{g} and commitment decisions \mathbf{z} , i.e.,

$$X := \{(\mathbf{g}, \mathbf{z}) \mid m_j z_j \leq g_j \leq M_j z_j, z_j \in \{0, 1\}, \forall j\}.$$

The partial Lagrangian dual problem of (11) is formulated as follows:

$$\min_{\lambda} \varphi(\lambda), \quad (12)$$

where

$$\begin{aligned} \varphi(\lambda) := & \max_{d \geq 0} \{u(d) - \lambda d\} \\ & + \max_{(\mathbf{g}, \mathbf{z}) \in X} \left\{ \lambda \sum_j g_j - \left\{ \sum_j C_j(g_j) + \sum_j S_j z_j \right\} \right\}. \end{aligned} \quad (13)$$

Note that our model takes into account price-sensitive demands and hence is different from existing CHP models such as those in [6], [10], [11]. Now we have reached the following result.

Proposition 3: The following statements hold:

- 1) The partial Lagrangian dual problem (12) has an optimal solution $(\lambda^*, \mathbf{g}^*, \mathbf{z}^*, d^*)$.
- 2) The problem (12) is equivalent to

$$\max_{d \geq 0} u(d) - v^{cc}(d),$$

where the v^{cc} is the biconjugate of v (i.e., the convex conjugate of the convex conjugate of v).

- 3) λ^* is a convex hull price, i.e.,

$$\lambda^* \in \partial v^h(d^*).$$

Proof: The idea behind the proof of 1) is to use the general version of Weierstrass' Theorem [12], which states that φ has a minimum point if φ is a closed² proper³ function and has a nonempty and bounded level set. From duality theory it is known that φ is lower semicontinuous, and this guarantees the closedness of φ . For all $\lambda < 0$, $\varphi(\lambda) = \infty$ holds since the first term in (13) is infinite, ∞ . For all $\lambda \geq 0$, the first term in (13) is finite, and an optimal solution exists as in (2). The second term in (13) is also finite, and an optimal solution exists because of the compactness of X . Thus, if λ^* exists, λ^* is a nonnegative number and $(\mathbf{g}^*, \mathbf{z}^*, d^*)$ also exists. (This implies that $d \leq \sum_j g_j$ is an effective constraint.) Let us choose $\gamma \in \mathbb{R}$ so that the level set $L = \{\lambda \mid \varphi(\lambda) \leq \gamma\} \in (0, \infty)$ is non-empty. From (13), we have

$$\varphi(\lambda) \geq u(0) + \lambda \sum_j M_j - (\sum_j C_j(M_j) + \sum_j S_j)$$

²A function f is said to be *closed* if $\text{epi}(f)$ is a closed set.

³A function f is said to be *proper* if there exists $\mathbf{x} \in \mathbb{R}^n$ such that $f(\mathbf{x}) \neq \infty$ and there does not exist $\mathbf{x}' \in \mathbb{R}^n$ such that $f(\mathbf{x}') = -\infty$.

for any λ . The right-hand side of the inequality is derived by setting $d = 0$, $g_j = M_j$, and $z_j = 1$ to (13). There exists a sufficiently large $\hat{\lambda} > 0$ such that $\varphi(\hat{\lambda}) > \gamma$. Hence, L is bounded, and the conditions on the existence of a minimum point of φ are satisfied.

The remaining parts can be obtained by mimicking the argument in [6]. The second term in (13) is written as

$$\begin{aligned} \max_{\eta, (\mathbf{g}, \mathbf{z}) \in X} \left\{ \lambda \eta - \left\{ \sum_j C_j(g_j) + \sum_j S_j z_j \right\} \mid \eta = \sum_j g_j \right\} \\ = \max_{\eta} \{ \lambda \eta - v(\eta) \} = v^c(\lambda), \end{aligned}$$

where v^c is the convex conjugate function of v . Problem (12) can be expressed as

$$\begin{aligned} \min_{\lambda} \varphi(\lambda) &= \min_{\lambda} \max_{d \geq 0} \{u(d) - \lambda d + v^c(\lambda)\} \\ &= \max_{d \geq 0} \min_{\lambda} \{u(d) - \lambda d + v^c(\lambda)\} \\ &= \max_{d \geq 0} \{u(d) - v^{cc}(d)\}, \end{aligned}$$

where v^{cc} is the biconjugate function of v , i.e., the convex conjugate function of v^c . The second equality holds from [12, Prop. 2.6.4]. To prove 3), we obtain the following result for all $d \geq 0$:

$$\begin{aligned} v^{cc}(d) &= \sup_{\lambda} \{d\lambda - v^c(\lambda)\} \\ &\geq d\lambda^* - v^c(\lambda^*) \\ &= d\lambda^* - v^c(\lambda^*) + \lambda^* d^* - \lambda^* d^* \\ &= v^{cc}(d^*) + \lambda^*(d - d^*). \end{aligned}$$

This implies that λ^* is a subgradient of v^{cc} at d^* . Since $v^{cc} = v^h$ holds (see [12]), this completes the proof. ■

Proposition 3 implies that λ^* minimizes the uplift payment with the demand d^* . Thus, it is reasonable to choose λ^* as a price.

V. SUBGRADIENT METHOD

The algorithms presented in [10], [11] cannot be applied to (12) because of the difference between the pricing models. In this section, therefore, we describe a subgradient pricing algorithm for (12). Recall that the dual function $\varphi : \mathbb{R} \rightarrow (-\infty, \infty]$ is defined as (13). φ can be decomposed into independent maximizing functions:

$$\begin{aligned} \varphi(\lambda) &= \varphi_d(\lambda) + \varphi_s(\lambda), \\ \varphi_d(\lambda) &= \max_{d \geq 0} u(d) - \lambda d, \end{aligned} \quad (14)$$

$$\varphi_s(\lambda) = \max_{(\mathbf{g}, \mathbf{z}) \in X} \lambda \sum_j g_j - \left\{ \sum_j C_j(g_j) + \sum_j S_j z_j \right\} \quad (15)$$

The subgradient of φ at $\hat{\lambda}$ is given by

$$\sum_j g_j(\hat{\lambda}) - d(\hat{\lambda}) \in \partial \varphi(\hat{\lambda}), \quad (16)$$

where $d(\hat{\lambda})$ (or $g_j(\hat{\lambda})$, for all j) is an optimal solution of (14) (or (15)), with $\lambda = \hat{\lambda}$ (see [12]). Since (14) and (15) corresponds to problem (1) and (3), respectively, the ISO can make the consumer and supplier solve (14) and (15), respectively, by sending $\hat{\lambda}$ as a price. This allows the ISO to obtain $\sum_j g_j(\hat{\lambda})$ and $d(\hat{\lambda})$, and hence, it can use subgradient methods to solve (12) in the electricity market.

The following is an iterative algorithm for solving (12).

Algorithm 1: Set the step size $\gamma^k > 0$ ($k = 1, 2, \dots, N$).

- 1) The ISO sets the initial price λ^0 .
- 2) According to the given price λ^k , the consumer and supplier adjust their respective demand and supply plans by using the following update steps:

$$d^k \in \arg \max_{d \geq 0} u(d) - \lambda^k d,$$

$$(g^k, z^k) \in \arg \max_{(g, z) \in X} \lambda^k \sum_j g_j - \{\sum_j C_j(g_j) + \sum_j S_j z_j\}.$$
- 3) The ISO updates the price as

$$\lambda^{k+1} = \lambda^k + \gamma^k (d^k - \sum_j g_j^k).$$
- 4) Repeat steps 2 and 3 N times. The ISO then fixes the price $\lambda = \lambda^N$. The scheduled levels of production and load are $d = d^N$.

It is proved in [12] that the subgradient method with an appropriate step size generates a sequence $\{\lambda^k\}$ which converges to an optimal solution λ^* . We can obtain the necessary and sufficient optimality conditions for (12) by modifying the optimality conditions of (9) in [10]. However, to check these optimality conditions, we have to solve an additional convex quadratic program [10, eq.(12)] at each iteration. In addition, our algorithm requires the supplier to solve a mixed integer problem in Step 2 every iteration. These calculation costs may be high. In the following section, we demonstrate that our algorithm reaches sufficiently small uplift payments in a realistic number of iterations without the ISO having to check the optimality conditions.

VI. NUMERICAL SIMULATIONS

A. Problem settings

Here, we present numerical simulations that confirm the efficiency of our pricing model and algorithm in reducing uplift payments. We assume that the ISO makes the next day's pricing decision hourly. We shall use the example of Gribik et al. [6] as shown in the following table.

TABLE I. EXAMPLE OF THREE GENERATORS IN [6]

	Capacity(MW)	Generators		
		A	B	C
Fixed Cost(\$)		0	6000	8000
Var cost1(\$/MW)	100	65	40	25
Var cost2(\$/MW)	100	110	90	35

The numerical algorithms were written in R language version 3.0.0. The GNU Linear Programming Kit package of version 0.3-10 was used for solving the linear programming and mixed integer programming problems.

Figure 2 illustrates four different cost functions. 'UCP' means the minimum cost function $v(y)$ of (9). 'Dispatchable' means the optimal value function $v^d(y)$ of the continuous relaxation model of UCP; i.e., the model replaces the 0-1 integer constraints $z_j \in \{0, 1\}$ in (9) by $0 \leq z_j \leq 1$ for all j . The Dispatchable pricing model in [6] uses the marginal cost of $v^d(y)$ as a price. We used this model in the comparison

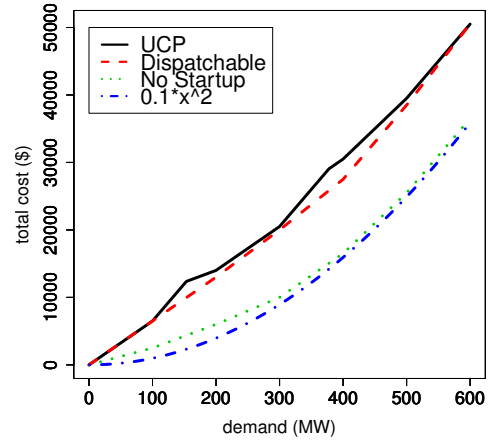


Fig. 2. Cost functions.

of the uplift payments. 'No Startup' means the minimum cost function that ignores the startup cost of the generators. Furthermore, we used the convex cost function $v^q(y) := 0.1y^2$ as a quadratic approximation to No Startup.

Following [4], we used the hourly demand function,

$$D_t(\lambda) = \mu_1 d_{1,t} + \mu_2 (1 + \delta_{2,t}) d(\lambda), \quad (t = 1, 2, \dots, 24),$$

where μ_1, μ_2 are positive parameters, $d_{1,t}$ is a positive constant, and $\delta_{2,t}$ is a random variable distributed with $\mathcal{N}(0, 0.01^2)$. The first term represents the minimum necessary demand, and the second term represents the swing in demand depending on prices. We used actual hourly demand data of the Tokyo Electric Power Company from 0:00 to 23:00 August 30, 2012 [13] as $d_{1,t}$ ($t = 1, 2, \dots, 24$). μ_2 was arbitrarily fixed in order to take account of price elasticity. μ_1 was adjusted so that the sum of simulated hourly demands $D_t(\lambda_t^N)$ remained nearly equal to the sum of the scaled actual hourly demands $d_{1,t}/100$ ($t = 1, 2, \dots, 24$), i.e.,

$$\sum_{t=1}^{24} D_t(\lambda_t^N) \approx \frac{1}{100} \sum_{t=1}^{24} d_{1,t}.$$

We defined a logarithmic utility function u with a positive parameter a :

$$u(d) = a \log(d).$$

a was adjusted so that the sum of $d(\lambda_t^N) = \max\{0, \{d \mid \dot{u}(d) = \lambda_t^N\}\}$ as in (2) remained nearly equal to the sum of the scaled actual hourly demands $d_{1,t}/100$ ($t = 1, 2, \dots, 24$), i.e.,

$$\sum_{t=1}^{24} d(\lambda_t^N) \approx \frac{1}{100} \sum_{t=1}^{24} d_{1,t}.$$

We set the parameters as follows: $N = 100$, $a = 3.9 \times 10^4$, $\mu_1 = 0.008$ and $\mu_2 = 0.2$.

B. Simulation results

First, let us compare the uplift payments of our model with the convex cost models in [3], [4]. The convex cost model uses $v^q(y)$ as cost function, and its electricity price is given by a locational marginal price (LMP). The results are summarized in Fig. 3. The simulation results for 24 hours are plotted as points. Each point shows the relation between the

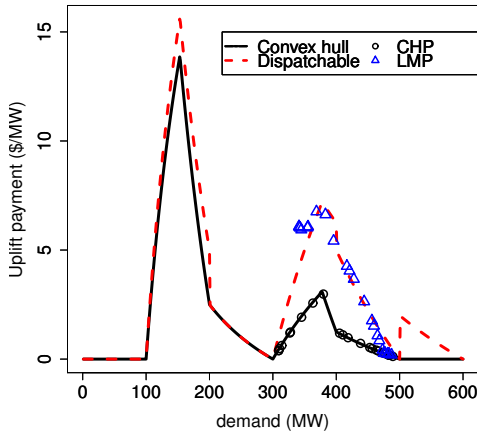


Fig. 3. Comparison of uplift payments of CHPs and LMPs with the convex cost function $0.1x^2$.

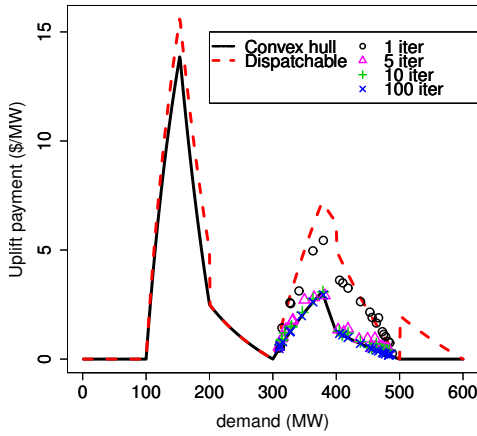


Fig. 4. Uplift payments at each iteration.

demand $D_t(\lambda_t^*)$ and uplift payment $\Pi(\lambda_t^*, D_t(\lambda_t^*))$ for hourly optimal prices λ_t^* ($t = 1, 2, \dots, 24$). The optimal prices λ_t^* for our model were calculated with (12), whereas optimal prices for the existing convex cost model were calculated as in [4]. The solid (dashed) line illustrates the uplift payments with the convex hull model (dispatchable model). In the existing convex cost model, the uplift payments accrue as much as in the dispatchable model. On the other hand, the uplift payments determined by our model are less than half that of the existing model.

Next, let us investigate the behavior of the subgradient algorithm. As we noted in Section V, the existing algorithms cannot be applied to our model. Thus, we can only show results for our algorithm. We choose the step size $\gamma^k = 1/10k$ ($k = 1, 2, \dots, N$) and initial price $\lambda_t^0 = 100[\$]$ ($t = 1, 2, \dots, 24$). Fig. 4 shows the uplift payments versus demand D_t for the price after the k -th iteration λ_t^k ($t = 1, 2, \dots, 24$, $k = 1, 5, 10, 100$). We can see that the subgradient method generates a sequence which reaches sufficiently small uplift payments in a few iterations. Consequently, although our market model requires the supplier to solve a mixed integer problem every iteration, it may not be much of a disadvantage.

VII. CONCLUSION

We developed an electricity market model that takes into account both the demand response and startup costs of generators. Because of the startup costs, uplift payments, which are disadvantageous to the supplier, may occur. To reduce uplift payments, we devised a pricing model and algorithm based on a subgradient method. We theoretically proved that our pricing model produces the convex hull price, which minimizes the uplift payment for an equilibrium demand. Numerical results showed the advantage of our pricing model over existing locational marginal pricing models with convex cost functions. Our pricing algorithm generated electricity prices that entail sufficiently small uplift payments in a realistic number of iterations.

We are planning to extend our model to a multi-agent and multi-period one with network constraints as in [5].

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REFERENCES

- [1] M. Albadi and E. El-Saadany, "A summary of demand response in electricity markets," *Electric Power Systems Research*, vol. 78, no. 11, pp. 1989–1996, Nov. 2008.
- [2] New York ISO, "NYISO (About NYISO – understanding the market – the energy market)," http://www.nyiso.com/public/about_nyiso/understanding_the_markets/energy_market/index.jsp, accessed: 2013-05-11.
- [3] M. Roozbehani, M. A. Dahleh, and S. K. Mitter, "Volatility of power grids under real-time pricing," *IEEE Transactions on Power Systems*, vol. 27, no. 4, pp. 1926–1940, Nov. 2012.
- [4] Y. Miyano and T. Namerikawa, "Load leveling control by real-time dynamical pricing based on steepest descent method," in *SICE Annual Conference*, Akita University, Akita, Japan, 2012, pp. 131–136.
- [5] W. Hogan and B. Ring, "On minimum-uplift pricing for electricity markets," Harvard University, Tech. Rep., 2003. [Online]. Available: http://www.hks.harvard.edu/fs/whogan/minuplift_031903.pdf
- [6] P. R. Gribik, W. W. Hogan, and S. L. Pope, "Market-clearing electricity prices and energy uplift," Tech. Rep., 2007. [Online]. Available: http://www.hks.harvard.edu/fs/whogan/Gribik_Hogan_Pope_Price_Uplift_123107.pdf
- [7] M. Bjørndal and K. Jörnsten, "Equilibrium prices supported by dual price functions in markets with non-convexities," *European Journal of Operational Research*, vol. 190, no. 3, pp. 768–789, Nov. 2008.
- [8] B. Zhang, P. B. Luh, and E. Litvinov, "On reducing uplift payment in electricity markets," in *Power Systems Conference and Exposition, 2009. PSCE '09. IEEE/PES*, Mar. 2009.
- [9] R. P. O'Neill, P. M. Sotkiewicz, B. F. Hobbs, M. H. Rothkopf, and W. R. Stewart, "Efficient market-clearing prices in markets with nonconvexities," *European Journal of Operational Research*, vol. 164, no. 1, pp. 269–285, Jul. 2005.
- [10] G. Wang, U. V. Shanbhag, T. Zheng, E. Litvinov, and S. Meyn, "An extreme-point subdifferential method for convex hull pricing in energy and reserve markets—Part I: Algorithm structure," *IEEE Transactions on Power Systems*, vol. PP, no. 99, 2013, early access articles, DOI: 10.1109/TPWRS.2012.2229302.
- [11] C. Wang, P. B. Luh, G. Paul, L. Zhang, and T. Peng, "The subgradient-simplex based cutting plane method for convex hull pricing," in *Power and Energy Society General Meeting, 2010 IEEE*, Jul. 2010.
- [12] D. P. Bertsekas, A. Nedic, and A. E. Ozdaglar, *Convex Analysis and Optimization*, ser. Optimization and Computation Series. Athena Scientific, 2003.
- [13] Tokyo Electric Power Company (TEPCO), "TEPCO electricity forecast," <http://www.tepco.co.jp/en/forecast/html/index-e.html>, accessed: 2013-04-29.