Energy saving by Extremum-Seeking Control using an Actuator with Adjustable Stiffness (AwAS)

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Abstract: In repetitive motions, Actuator with Adjustable Stiffness (AwAS) can realize resonance and largely reduce energy consumption thanks to its ability to change the level of stiffness and adjust its natural frequency to the desired motion's frequency. In conventional method, an ideal stiffness value is calculated from frequency of the target motion and parameters of AwAS beforehand. This is an off-line system. Extremum-seeking control allows AwAS to optimize its stiffness on-line. Therefore, even if the frequency of the desired motion or parameters of AwAS change in unexpected time, it can optimize the stiffness and realize energy saving continuously. It was confirmed by numerical simulations.

Keywords: energy saving, adjustable stiffness, resonance, extremum-seeking control.

1. INTRODUCTION

Recently, industrial robots came to be seen in various situations. Thanks to these robots, people don't have to do dangerous works and mass production was realized because of high work efficiency. But, a new problem is occurring. It's an energy problem. In order to operate the robots, large amount of energy is required. So when it comes to running them for a long time, the amount of energy consumed is immeasurable. Therefore, reducing the energy consumption of the robots, that is "energy saving" attracts attention.

Repetitive motions are common movements of robots. If an actuator gives all energy to them to achieve these motions, a lot of energy will be consumed. Energy saving means to reduce the energy consumption of the actuator, specifically, converts potential energy into kinetic energy and uses it for achieving the motion instead of the energy of the actuator. Familiar examples of this are pendulum motion and human walking. Pendulums use gravitational potential energy as kinetic energy and can continue periodic motion without external force. Also it is known that humans use muscles and tendons efficiently and are walking in energy conservation. With reference to these phenomena, the research of energy saving, such as the passive dynamic walker [1, 2], is actively conducted.

In this paper, energy saving using an Actuator with Adjustable Stiffness (AwAS) [3-5] is presented. AwAS is an actuator which the elastic element is attached to. Using this actuator, the robots have a natural frequency. If the natural frequency matches the frequency of the target periodic motion, the resonance phenomenon will be derived. When resonance is realized, theoretically, the target motion can be achieved without the energy of the actuator because there is enough energy (potential energy) thanks to the elastic element [6, 7]. With this approach, considerable energy saving is expected.

Moreover, in this study, "extremum-seeking control" [8] is applied to the control system. It allows AwAS to optimize its stiffness on-line. Thus, even in case the frequency of the target motion or parameters of AwAS are time-varying, it is able to make stiffness ideal value and continue achieving energy saving. It was shown by numerical simulations. To evaluate energy efficiency, an energy cost function was used.

From these results, we expect that AwAS can save energy even when the parameters of it are unknown or the optimize stiffness value is impossible to be calculated because the desired periodic motion is very complex.

The rest of this paper is structured as follows. In section 2, the design of AwAS is explained. Optimal stiffness value to realize resonance is calculated in section 3 and the conventional energy saving system using AwAS is shown in section 4. The principle of extremum-seeking is stated in section 5 and the optimal stiffness based on the theory of extremum-seeking is shown in section 6. The proposed system is shown in section 7. The numerical simulations are presented in section 8 while the conclusion and future work are discussed in section 9.

2. AWAS

AwAS has a structure which is shown in Fig.1.

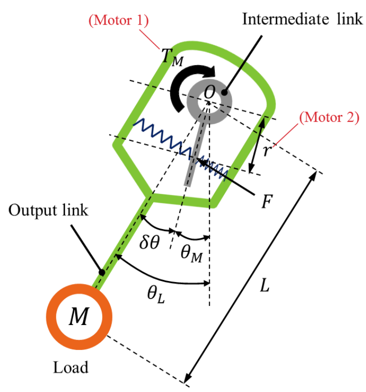

Fig. 1 AwAS

It consists of two links, the intermediate link and the output link, and two springs which are connected on one side to the intermediate link and on the other side to the output link. Furthermore, two motors are attached to AwAS. The motor 1 (M1) can rotate the intermediate link, and the motor 2 (M2) can move the position of the springs (control the length of the lever arm) to regulate the stiffness of AwAS. At first, we explain the mechanism to change the stiffness of AwAS. When the position of the links is as shown in Fig.1, the resultant force from the elastic elements is given by

$$\begin{aligned} F &= K_s(p+\delta X) - K_s(p-\delta X) \\ &= 2K_s\delta X = 2K_s r \sin\delta\theta. \end{aligned} \tag{1}$$

p is the spring's pretension, δx is the spring's deflection, and K_s is the spring constant of the elastic elements. The resultant torque applied to the intermediate link by the force is calculated as

$$T = Fr\cos\delta\theta = 2K_s r^2 \sin\delta\theta\cos\delta\theta. \tag{2}$$

So, the stiffness of AwAS can be shown by partial differentiation of this torque with respect to the angular deflection $\delta\theta$

$$K = \frac{\partial T}{\partial(\delta\theta)} = 2K_s r^2(2\cos^2\delta\theta - 1). \tag{3}$$

From Eq.(3) , it is found that AwAS can change the stiffness by regulating the position of the springs r.

AwAS can be physically modeled as shown in Fig. 2.

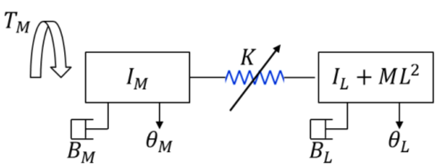

Fig. 2 Physical model of AwAS

Then, the motion equations of AwAS are given as

$$I_M\ddot{\theta}_M + B_M\dot{\theta}_M + K(\theta_M - \theta_L) = T_M. \tag{4}$$

$$(I_L + ML^2)\ddot{\theta}_L + B_L\dot{\theta}_L + K(\theta_L - \theta_M) = 0. \tag{5}$$

Eq.(4) and Eq.(5) mean the motion equation of the intermediate link and the output link respectively. Each parameter of these equations and Fig.1 is described in Table 1.

3. OPTIMAL STIFFNESS (THEORY OF RESONANCE)

When the target motion is a periodic motion and its frequency f is constant, the resonance phenomenon will be able to be realized if the AwAS's natural frequency f_n matches f. The stiffness of AwAS which lead to resonance is denoted as the optimal stiffness value K_d in this time. K_d is calculated as follows.

Table 1 Description of the parameters

I_M	inertia of the intermediate link
I_L	inertia of the output link
B_M	damping at the intermediate link side
B_L	damping at the output link side
θ_M	position of the intermediate link
θ_L	position of the output link
$\delta\theta$	angular deflection
T_M	motor torque
K	joint stiffness
M	mass attached to the output link
L	output link length
O	axis of rotation
r	position of the springs (lever arm)

To begin with, calculate f_n (simplify the calculation, B_L is assumed not to exist). It is shown as

$$f_n = \frac{1}{2\pi}\sqrt{\frac{K}{(I_L + ML^2)}}. \tag{6}$$

To derive resonance, f_n should be equal to f. Thus, K_d is given by

$$K_d = 4\pi^2 f^2 (I_L + ML^2). \tag{7}$$

The springs' position which realize K_d is designated the optimal position r_d, and it is shown as

$$r_d = \sqrt{\frac{K_d}{2K_s(2\cos^2\delta\theta - 1)}}. \tag{8}$$

4. ENERGY SAVING WITH AWAS (CONVENTIONAL METHOD)

The conventional control system is shown in Fig.3. The desired motion θ_L and the optimal stiffness K_d calculated from θ_L are used as external signals.

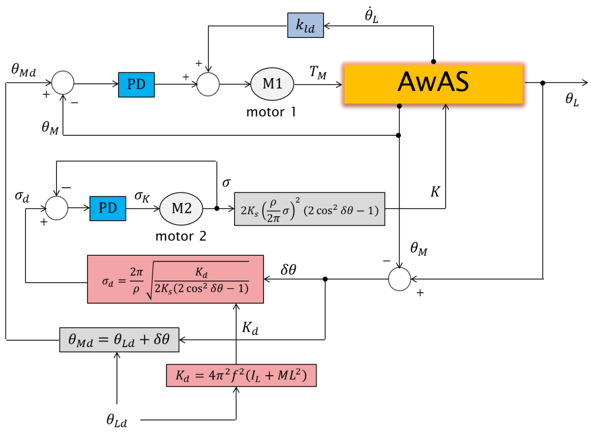

Fig. 3 Schematic diagram of the control system

In the conventional method, it is necessary that the frequency of the motion and the all parameters of AwAS are known.

The parameter σ represents the rotation angle of the motor 2. As stated in section 2, r is regulated by the motor 2. Specifically, the rotary motion is converted into the linear motion by the ball screw and the springs move. The relation between σ and r is

$$\sigma = \frac{2\pi}{\rho} r. \tag{9}$$

Where ρ denotes the pitch of the ball screw. Then, the optimal rotation angle of the motor 2 σ_d is calculated as

$$\sigma_d = \frac{2\pi}{\rho} \sqrt{\frac{K_d}{2K_s(2\cos^2\delta\theta - 1)}}. \tag{10}$$

Each motor is controlled by the PD controller. The motor 1 has an additional active damping term $k_{ld}\dot{\theta}_L$ to attenuate the link oscillation, so the reference signal of the motor 1 T_M is given by

$$T_M = k_{mp}(\theta_{Md} - \theta_M) + k_{md}(\dot{\theta}_{Md} - \dot{\theta}_M) + k_{ld}\dot{\theta}_L. \tag{11}$$

And the reference signal of the motor 2 is

$$\sigma_K = k_p(\sigma_d - \sigma) + k_d(\dot{\sigma}_d - \dot{\sigma}). \tag{12}$$

Using this system, AwAS can derive the resonance and achieve the target motion with a little energy. But, when the desired motion's frequency and AwAS's parameters are subject to change, AwAS can't adjust its stiffness to the optimal value on-line. Therefore, AwAS isn't able to continue to derive the resonance and realize energy-saving.

The concrete example is shown in Figs.4 to 6. The desired motion is $\theta_{Ld} = 0.2\sin(2\pi f t)$[rad]. At first, the AwAS began to oscillate with $f = 2.5$[Hz], $M = 1.0$[kg]. After 50 seconds, f was changed to 4.0[Hz]. After another 50 seconds, M was changed to 2.0[kg].

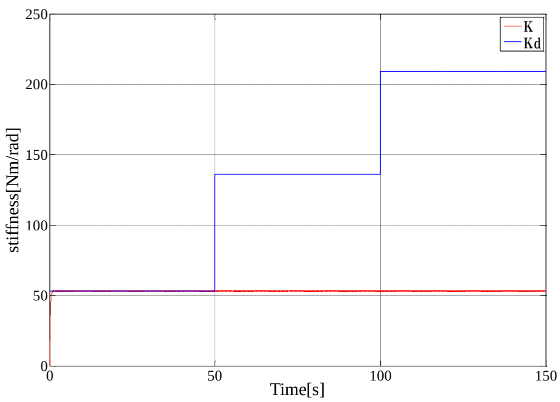

Fig. 4 Stiffness of AwAS K(red) and optimal stiffness K_d(blue)

As shown in Fig.4, the stiffness of AwAS isn't optimized when f or M changes. For this reason, the intermediate link oscillates widely so AwAS uses much energy (Fig.5). Furthermore, from Fig.6, it is known that the desired motion isn't achieved after 50 seconds.

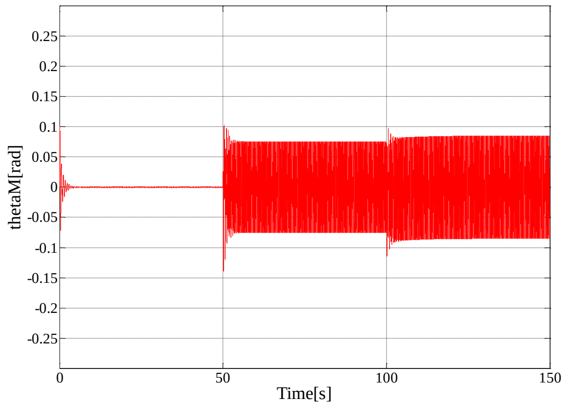

Fig. 5 Intermediate link trajectory θ_M

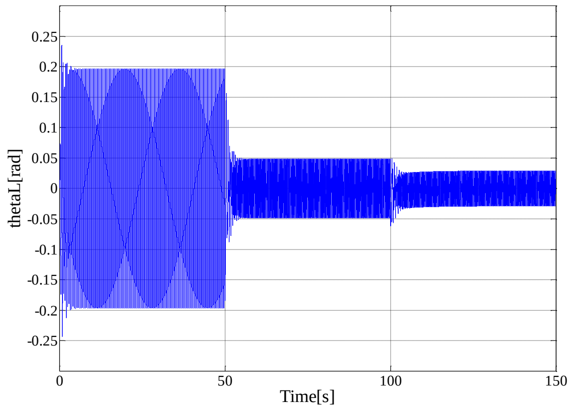

Fig. 6 Output link trajectory θ_L

5. EXTREMUM SEEKING CONTROL

Extremum-seeking control has a structure shown in Fig.7. It adjusts the parameter to make the cost function J minimum or maximum value continuously. In this case, the parameter is σ.

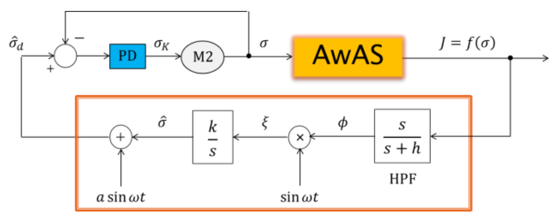

Fig. 7 Extremum-seeking control

Then, we state the theory of extremum-seeking control. At first, we posit the form of the cost function as

$$J = f(\sigma) = f^* + \frac{f''}{2}(\sigma - \sigma^*)^2. \tag{13}$$

Where f^* and σ^* represents the extremum of the cost function J and the optimal value of the parameter σ. When $f'' > 0 (f'' < 0)$, J is a downwardly(upwardly) convex function.
From Fig.7,

$$\sigma = \hat{\sigma_d} = a\sin\omega t + \hat{\sigma}. \tag{14}$$

When substituted into Eq.(13), gives

$$\begin{aligned} J &= f^* + \frac{f''}{2}(a\sin\omega t + \hat{\sigma} - \sigma^*)^2 \\ &= f^* + \frac{f''}{2}(a\sin\omega t - \tilde{\sigma})^2. \qquad (\tilde{\sigma} = \sigma^* - \hat{\sigma}) \end{aligned} \tag{15}$$

Expanding this expression further, calculated as

$$\begin{aligned} J &= f^* + \frac{f''a^2}{2}\sin^2\omega t - f''a\tilde{\sigma}\sin\omega t + \frac{f''}{2}\tilde{\sigma}^2 \\ &= f^* + \frac{f''a^2}{4} - \frac{f''a^2}{4}\cos 2\omega t - f''a\tilde{\sigma}\sin\omega t + \frac{f''}{2}\tilde{\sigma}^2. \end{aligned} \tag{16}$$

The high pass filter removes low frequency components

$$\begin{aligned} \phi &= \frac{s}{s+h}[J] \\ &\approx -\frac{f''a^2}{4}\cos 2\omega t - f''a\tilde{\sigma}\sin\omega t + \frac{f''}{2}\tilde{\sigma}^2. \end{aligned} \tag{17}$$

Then, ξ is given by

$$\xi = -\frac{f''a^2}{4}\cos 2\omega t\sin\omega t - f''a\tilde{\sigma}\sin^2\omega t + \frac{f''}{2}\tilde{\sigma}^2\sin\omega t. \tag{18}$$

Applying the identity

$$\cos 2\omega t\sin\omega t = \frac{\sin 3\omega t - \sin\omega t}{2}.$$

ξ is calculated as

$$\begin{aligned} \xi = &-\frac{f''a^2}{8}(\sin 3\omega t - \sin\omega t) - \frac{f''a}{2}\tilde{\sigma} \\ &+ \frac{f''a}{2}\tilde{\sigma}\cos 2\omega t + \frac{f''}{2}\tilde{\sigma}^2\sin\omega t. \end{aligned} \tag{19}$$

There are two conditional expressions

$$\hat{\sigma} = \frac{k}{s}[\xi] \quad \Leftrightarrow \quad \dot{\hat{\sigma}} = k\xi. \tag{20}$$

$$\dot{\tilde{\sigma}} = \dot{\sigma^*} - \dot{\hat{\sigma}} = -\dot{\hat{\sigma}}. \tag{21}$$

Considering these conditions and that the first, third, and forth term of Eq.(19) are attenuated by an integrator, getting

$$\dot{\tilde{\sigma}} = -k\xi = \frac{kf''a}{2}\tilde{\sigma}. \tag{22}$$

Since $kf''a < 0$, this is a stable system. Thus, $\tilde{\sigma}$ converges to 0 as time passes. In terms of the original problem, $\hat{\sigma}$ converges to σ^*, so σ is kept within a small distance of σ^*. As a result, the cost function converges to around their extremum f^*.

6. OPTIMAL STIFFNESS (THEORY OF EXTREMUM-SEEKING)

In this paper, the kinetic energy of the intermediate link is used as a cost function.

$$J_1 = f(\sigma) = \int_0^t \frac{1}{2} I_M \dot{\theta}_M^2 dt. \tag{23}$$

To save energy means to reduce the energy consumption of the motor 1, this is consistent with decreasing the oscillation of the intermediate link. When the desired motion is periodic, the parameter σ which minimize this cost function is equal to which is obtained from the concept of resonance (Eq.(10)). This means the proposed method allows AwAS to derive resonance on-line.

proof

From Eqs.(4) and (5), the relational expression between the intermediate link and the output link is

$$\theta_M = \frac{I_L + ML^2}{K}\ddot{\theta}_L + \theta_L. \tag{24}$$

Accordingly, the cost function Eq.(23) is modified as

$$\begin{aligned} J_1 = \frac{1}{2} I_M \int_0^t \Bigg\{ &\left(\frac{I_L + ML^2}{K}\right)^2 \dddot{\theta}_L{}^2 \\ &+ \frac{2(I_L + ML^2)}{K}\dddot{\theta}_L\dot{\theta}_L + \dot{\theta}_L{}^2 \Bigg\} dt. \end{aligned} \tag{25}$$

Assuming the desired motion is a single frequency sine wave

$$\theta_{Ld} = A\sin(\omega t + \phi). \tag{26}$$

Then, the control system is a linear system, so the output θ_L is given by

$$\theta_L = B\sin(\omega t + \psi). \tag{27}$$

Substitute the constant stiffness K_c and Eq.(27) into Eq.(25), calculated as

$$\begin{aligned} J_1 = \frac{1}{2} I_M B^2\omega^2 \Bigg\{ &\left(\frac{I_L + ML^2}{Kc}\right)^2 \omega^4 \\ &- \frac{2\left(I_L + ML^2\right)}{K_c}\omega^2 + 1 \Bigg\} \int_0^t \cos^2(\omega t + \psi) dt. \end{aligned} \tag{28}$$

The optimal stiffness value K_{opt} which minimize Eq.(28) is K_c which satisfies the equation

$$\begin{aligned} \frac{\partial J_1}{\partial K_c} = \frac{1}{2} I_M B^2\omega^2 \Big\{ &(-2)\left(I_L + ML^2\right)^2 \omega^4 {K_c}^{-3} \\ &+ 2\left(I_L + ML^2\right)\omega^2 {K_c}^{-2} \Big\} \int_0^t \cos^2(\omega t + \psi) = 0. \end{aligned} \tag{29}$$

Therefore, from Eq.(29), K_{opt} is calculated as

$$\begin{aligned} K_{opt} &= \left(I_L + ML^2\right)\omega^2 \\ &= 4\pi^2 f^2 \left(I_L + ML^2\right). \end{aligned} \tag{30}$$

Eq.(30) equal to Eq.(7), so to minimize Eq.(23) means to derive resonance.

7. ENERGY SAVING WITH AWAS (EXTREMUM-SEEKING CONTROL)

The proposed system incorporating extremum- seeking control is shown in Fig.8. Compared with the conventional method, the AwAS's optimal stiffness value K_d is removed from the external signals. This is because the extremum-seeking control enables AwAS to optimize its stiffness on-line. This is the main advantage of new system.

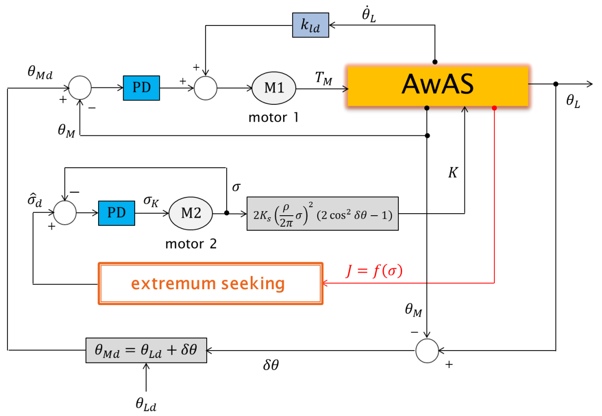

Fig. 8 Schematic diagram of the proposed system

The need to calculate K_d is gone, so we consider that it is possible to achieve energy-saving even the case that we can't calculate the optimal stiffness value because the target periodic motion is very complex, or the parameters of AwAS is unknown.

8. SIMULATION RESULTS

Numerical simulations are conducted to show the validity of the proposed method.

The parameters of AwAS are shown in Table 2.

Table 2 Value of the parameters

I_M	0.35[kgm^2]
I_L	0.1[kgm^2]
L	0.34[m]
K_s	16[N/mm]
ρ	0.01[m]

The damping of each link B_M and B_L are assumed to be negligible. The parameters used in extremum-seeking control are $a = -0.01$, $\omega = 1.0$, $k = 80$, $h = 1.0$, respectively. The target motion and the simulation condition are the same as in section 4. The results are shown in Figs.9 to 11.

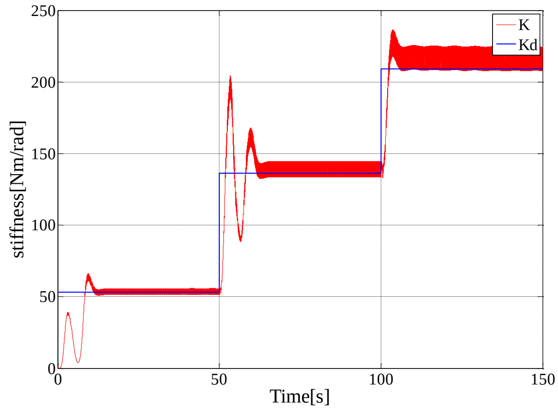

Fig. 9 Stiffness of AwAS K(red) and optimal stiffness K_d(blue)

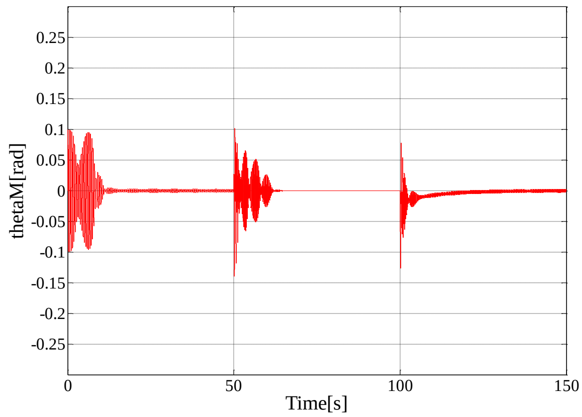

Fig. 10 Intermediate link trajectory θ_M

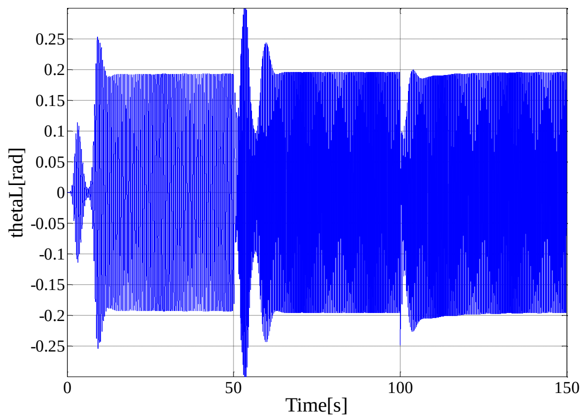

Fig. 11 Output link trajectory θ_L

From Fig.9, it can be seen that the proposed method allowed AwAS to optimize its stiffness on-line. It wasn't realized by the conventional method. In addition, from Figs.5 to 6 and 10 to 11, it is known that the trajectory of each link is obviously different. In the proposed method, by optimizing the stiffness, the oscillation of the intermediate link was suppressed and the output link achieved the desired motion continuously. It means that AwAS realized the target motion with a little energy in all conditions.

Furthermore, to evaluate these results quantitatively, we introduce the cost function as follows.

$$C = \int_{t_0}^{t} (K_d - K)^2 dt. \tag{31}$$

This expression indicates a gap between the optimal stiffness value and an actual AwAS's stiffness value. When C is small, the gap is small too. The values of C are summarized in table 3.

Table 3 Value of C

Time[s]	conventional ($\times 10^3$)	proposed ($\times 10^3$)
0˜50	0.4068	12.35
50˜100	344.2	17.91
100˜150	1217	11.57
total	1561	41.83

From this list, the effectiveness of the proposed method is evident. Table 4 shows the energy consumption in two cases, using a common actuator and using AwAS.

Table 4 Energy consumption in steady state

conditions	common actuator [W]	AwAS [W]
$f = 2.5$, $M = 1.0$	11.98	5.660
$f = 4.0$, $M = 1.0$	58.48	46.42
$f = 4.0$, $M = 2.0$	120.1	111.9

According to this table, it is clear that AwAS used less energy than a common actuator. Depending on the conditions, AwAS achieved energy saving up to 53%.

9. CONCLUSION

In this paper, energy-saving using AwAS was presented. When the desired motion is a periodic motion, AwAS can adjusts its stiffness and derive resonance. When resonance is derived, almost all energy which is needed to achieve the desired motion is generated by the elastic elements so the actuator uses little energy. In the conventional method, AwAS can realize energy-saving only when the condition (the target motion's frequency or the parameter of AwAS) is constant. But if the condition is changed, the system doesn't continue to save energy because it doesn't have an ability to optimize the stiffness of AwAS according to the condition. Therefore, we introduced "extremum-seeking control" into the system. It always adjusts the parameter to make the cost function minimum or maximum value constantly. Extremum-seeking control enables AwAS to optimize its stiffness on-line. So, even if the condition is changed, AwAS can continue to achieve energy-saving. These were shown by the numerical simulations. In future work, we will try to realize energy saving when the target motion is a complex (multi frequency) motion.

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